Towards classical de Sitter solutions in string theory

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ABSTRACT: We investigate the type II string effective potential at tree-level for de Sitter critical points. We derive necessary ingredients for having de Sitter solutions in orientifold models with fluxes. Furthermore, we examine some explicit O6 compactifications in IIA theory on manifolds with SU(3)-structure in the limit where the orientifold sources are smeared. In particular, we use a simple ten-dimensional Ansatz for four-dimensional de Sitter solutions and find the explicit criteria in terms of the torsion classes such that these de Sitter solutions solve the equations of motion. We have verified these torsion conditions for the cosets and the Iwasawa manifold and it turns out that the conditions cannot be fulfilled for these spaces. However this investigation allows us to find new non-supersymmetric AdS solutions for some cosets. It remains an open question whether there exist SU(3)-structure manifolds that satisfy the conditions on the torsion classes for the simple de Sitter solutions to exist.

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1 Introduction

The richness of string theory also presents us with a huge vacuum degeneracy problem. In lack of a dynamical principle to select a unique vacuum, there are two broad approaches one typically takes. In a bottom-up approach, one aims to construct models which realize as many known properties of our universe as possible. The rationale is that the criteria for a "realistic" solution may significantly reduce the space of vacua, and thus one can zero-in to a promising subset which hopefully points to *the* vacuum that describes our universe. Alternatively, one can quantify the "likelihood" of our universe by sampling the statistics of a vast number of vacua without imposing the prior that such vacua resemble the one in which we live. This latter approach is what underlines the idea of a string landscape, and has often been invoked to address the cosmological constant problem in string theory.

In contrast, the bottom-up approach has mainly been focussed on particle physics aspects without much reference to the cosmological constant. This is most apparent in local D-brane (and F-theory) model building where the requirement of having the low energy spectrum and interactions of the Standard Model or Grand Unified Theories puts non-trivial restrictions on *local* properties of the compactification, such as the types of singularities supported in the internal space. These bottom-up constraints are powerful in that they hold for a large class of models without having to fully specify the compactification details. Of course, they are only necessary conditions, as *global* constraints such as moduli stabilization and flux quantization can only be fully imposed with a specification of UV

completion. Nevertheless, they serve as a useful guide in the search for realistic vacua before a complete model is explicitly constructed.

Given the cosmological constant problem is a question that arises only in the context of quantum gravity, it should play an equally (if not more) important role in the selection of string vacua. A natural question is whether there are analogous bottom-up constraints on the underlying compactification in order for the resulting string theory solutions to have positive 4D energy density.[1] Naively, the answer is no since the cosmological constant is defined only after all moduli are stabilized and so details of compactifications are needed before this question can be addressed. As we shall see, however, under some assumptions which will be elaborated further, one can obtain a set of constraints on the internal manifold valid for a large class of models without specifying the compactification details. Our results thus suggest a different strategy to search for de Sitter solutions, allowing us to focus on promising regions of the landscape instead of constructing them in a model by model basis.

Our investigation is guided by various no-go theorems, some appeared in the recent literature [1–5] and some we proved along the way. We center our discussions on Type II string theories and their effective supergravity action since moduli stabilization is more developed in the Type II duality frames. In particular, it is well known by now that classical ingredients such as background fluxes have the effect of fixing moduli [6–10] (see e.g. [11–15] for reviews). Although non-perturbative effects are often invoked in scenarios of moduli stabilization (e.g., in the Type IIB context of [16]), the full moduli dependence of such effects is extremely difficult to determine explicitly. Therefore, much of the work on the subject amounts to demonstrating (by zero mode counting) that certain instanton effects crucial for moduli stabilization are non-vanishing, rather than providing an explicit computation of their magnitude and moduli dependence.

For ease of making our statements precise, we thus focus on finding de Sitter solutions with only *classical* objects such as fluxes, orientifold planes, and curvature along the lines of [1, 4, 5, 17, 18], since their contributions to the 4D potential are explicitly computable. In the "minimalist" spirit of [5], we do not consider introducing D-branes or orbifolding the internal manifold even though these ingredients also lead to a computable potential. This is because their presence also implies new moduli such as those arising from open strings and twisted sectors. In [1, 17] it has been argued that KK monopoles and NS5 branes lead to contributions in the 4D effective potential that can enhance the existence of de Sitter critical points. However, since our ultimate goal is to construct de Sitter solutions from a 10D point of view we refrain from introducing these objects since it is far from clear how the backreaction of such objects can be taken into account as to have a reliable 4D de Sitter solution. Of course there are still backreaction issues when one restricts to orientifolds, and admittedly we have only been able to find solutions in the smeared limit. It is nonetheless more likely that for configurations with just O-planes the backreaction can be computed and one would be able to tell whether the de Sitter solution still exists.

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[1] A conventional wisdom is to search for realistic vacua that preserve supersymmetry at the compactification scale, and that supersymmetry is dynamically broken (e.g., due to strong dynamics in the hidden sector) in the effective theory at lower energies. However, not all realistic features of the models (such as masses and couplings) necessarily persist after supersymmetry breaking and vacuum uplifting.

Within the framework of this "minimalist" approach there appeared some recent works on dS solutions in IIA [4, 5].[2] It is one of our aims to improve on these works since the proposed stable dS solution in [5] turns out to not solve the 10D equations of motion whereas the candidate example in [4] is perturbatively unstable. Furthermore, because of the complexity of the solution in [4] it is hard to check that it really solves the 10D equations of motion.[3]

We investigate the effective potential for such Type II compactifications and search for de Sitter critical points in models with orientifold sources and fluxes on a compact internal manifold. Our treatments for Type IIA and IIB theories are completely parallel except for some obvious changes as one goes between these duality frames. We derive several no-go conditions for the existence of de Sitter solutions, and explore some explicit models that circumvent them. In the specific case of SU(3)-structure manifolds in IIA with smeared O6 planes, we find de Sitter solutions that solve the 10D equations of motion when certain conditions on the torsion classes are satisfied, even though the stability of such de Sitter solutions needs to be checked once specific models are found. On the other hand, we verify that these torsion conditions are *not* satisfied for the coset geometries. These examples illustrate the utility and power of the no-go constraints. It remains an open problem whether there exist SU(3)-manifolds that satisfy the conditions on the torsion classes for these simple de Sitter solutions to be realized.

As an interesting aside we find that our analysis allows us to construct new non-supersymmetric AdS solutions for some coset geometries.

2 The coupling and volume dependence of V_{tree}

The number of scalar fields appearing in an effective 4D theory after compactification depends on the specifications of the compactification under consideration. Nonetheless there are 2 universal moduli that always appear, these are the string coupling ϕ and the internal volume $\mathcal{V}$. The appearance in the effective potential at tree-level is also universal, see for instance [1, 15]. In the following we re-derive these potential terms from type II supergravity since we will need these to derive our nogo theorems in the next section.

The metric Ansatz, in 10 dimensional string frame, that describes an unwarped reduction to $3+1$ dimensions is

$$\mathrm{d}s_{10}^2 = \tau^{-2}\mathrm{d}s_4^2 + \rho\,\mathrm{d}s_6^2\,, \tag{2.1}$$

where we have to take

$$\tau \equiv \rho^{3/2}\mathrm{e}^{-\phi}\,, \tag{2.2}$$

in order to find 4D Einstein frame.[4]

The NSNS fluxes are the H field strength and the metric flux. By metric flux we mean that the internal manifold has non-zero Ricci scalar. The energy contributions are

$$V_R = U_R\rho^{-1}\tau^{-2}\,, \qquad V_H = U_H\rho^{-3}\tau^{-2}\,, \tag{2.3}$$

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[2]For literature on non-classical dS solutions in IIA we refer to [19–21].

[3]In the sourceless case, that admits no dS solutions, there exist arguments showing that the dimensional reduction is consistent [22].

[4]In our conventions, the 10D string frame action is $\int \sqrt{|\,g\,|}\mathrm{e}^{-2\phi}(\mathcal{R} + 4(\partial\phi)^2 + \ldots)$.

where U_H denotes the integrated flux, $U_H = \int_6 H^2$, and U_R denotes minus the integrated curvature, $U_R = -\int_6 \mathcal{R}_6$. We will use similar notation in the following that we consider self-explanatory. For the RR q-form fluxes we find

$$V_q^{\mathrm{RR}} = U_q \tau^{-4} \rho^{3-q} \,. \tag{2.4}$$

For Dp and Op sources, with tension T_p, that fill the lower 4D spacetime and wrap a $(p-3)$-dimensional submanifold Σ we find

$$V_{Dp/Op} = T_p Vol(\Sigma) \tau^{-3} \rho^{\frac{p-6}{2}} \,. \tag{2.5}$$

This term is negative for O-planes and positive for D-branes.

From the above discussion we find that the form of the string effective potential in $D = 4$ at tree-level can be written as

$$V_{\mathrm{tree}} = a(\varphi)\tau^{-2} - b(\varphi)\tau^{-3} + c(\varphi)\tau^{-4} \,, \tag{2.6}$$

where φ denotes all scalars different from τ (including ρ).

In the case the internal space is unwarped and compact one easily verifies that the effective potential approach is correct since the $\partial_\rho V = 0 = \partial_\tau V$ equations correspond to specific linear combinations of the 10D dilaton equation of motion and the trace over the internal indices of the 10D Einstein equations as shown in appendix B.[5]

3 No-go theorems and minimal ingredients

In this section we consider all orientifold compactifications and focus on the form of the tree-level scalar potential. Since we require the O-planes to fill 4D space and wrap some internal submanifold, the Op-planes we consider have $p \geq 3$. If we furthermore insist that the O-planes do not break supersymmetry explicitly so that the resulting dS solutions correspond to supersymmetry breaking states in a supersymmetric theory, their dimensionality should differ by a multiple of 4. Finally, the O9-plane tadpoles are canceled by D9-branes which introduce open string moduli. With the minimalist approach we pursue here, we shall not consider this possibility though we expect our considerations can be applied to the O9 cases as well. Therefore, we end up with the following options in IIA: O4, O6, O8 and O4/O8 and in IIB: O3, O5, O7 and O3/O7.

The minimal ingredients. As originally discussed in [17], searching for de Sitter critical points with small vacuum energy of the potential corresponds to finding critical points of the quantity $4ac/b^2 \approx 1$ as a function of the other moduli φ:

$$\partial_\varphi \frac{4ac}{b^2} = 0 \,, \qquad \& \qquad \frac{4ac}{b^2} \approx 1 \,. \tag{3.1}$$

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[5]When warping is present one needs to be more careful in reducing the action. For instance, there exist models that allow de Sitter solutions without sources [23] (but with non-compact internal space), although the ρ, τ appearance in the naively reduced scalar potential would not allow for it.

With this simple result we can easily construct no-go conditions for dS solutions by investigating when $4ac/b^2$ allows for critical points. By focussing on just the ρ-dependence of $4ac/b^2$ we can give conditions that hold independently of the geometry and other ingredients specific to a model. In particular we will list the minimal ingredients that are necessary to have a critical point of $4ac/b^2$ for type IIA/B supergravity with sources.

For the single type Op reductions we have

$$\frac{4ac}{b^2} = \frac{\sum_q U_q \rho^{6-p-q}}{Vol_\Sigma^2 T_p^2} \left(U_R \rho^2 + U_H \right). \tag{3.2}$$

The demand that $4ac/b^2$ is stabilised close to 1 shows that

$$U_R \rho^2 + U_H > 0. \tag{3.3}$$

From $\partial_\rho(4ac/b^2) = 0$ we deduce that

$$2U_R \rho^{7-p} \sum_q U_q \rho^{-q} = -\left(U_R \rho^2 + U_H \right) \sum_q (6-p-q) U_q \rho^{5-p-q}. \tag{3.4}$$

This equation combined with (3.3) and the fact that U_H and U_q are all positive implies that for $p > 4$ we need to have $U_R > 0$ and hence we need negatively curved internal spaces. In general we deduce the following conditions from (3.4):

- O3 planes: When $U_R = 0$ we need at least U_H, F_1 and F_5. When $U_R \neq 0$ more possibilities arise.

- O4 planes: When $U_R = 0$ we need at least U_H, U_0 and U_q with $q > 2$. When $U_R \neq 0$ more possibilities arise.

- O5 planes: We minimally need positive U_R, U_1 and some other field strength turned on.

- O6 planes: The minimal conditions which were derived previously in [5] and are positive U_R, U_0 and some U_q with $q > 2$ (or positive U_R, U_0, U_H with U_q with $q > 0$.).

- O7 & O8 planes: We cannot stabilise $4ac/b^2$.

For the O4/O8 and O7/O3 setup the expressions for $\frac{4ac}{b^2}$ are more lengthy but a close look at the expressions shows that:

- O4/O8: One needs at least U_R and U_2, or U_0 and U_H.

- O3/O7: One needs at least U_R and U_3, or U_1 and U_H.

The above derivations use the dependence of the effective potential on ρ and τ which is equivalent to the 10D dilaton equation and traced internal Einstein equation in the smeared limit, as explained in appendix B. The traced external Einstein equation just fixes the value of the 4D cosmological constant, and contains no new information. But in some cases one is able to use some extra equation to find an extra relation. This was done in

GKP [10], where the F_5 equation of motion (or Bianchi identity) was used in the traced external Einstein equations to find extra nogo conditions. Let us briefly repeat the outcome of that result and furthermore drop the assumptions of [10] that the 4D space is Minkowski and that the internal space is a warped Calabi-Yau.

The Ansatz for F_5 in [10] is

$$F_5 = (1 + \star)\mathrm{d}\alpha \wedge \epsilon_4 \,, \tag{3.5}$$

where α is some function on the internal manifold (that is even under the O3 target space involution in case there is an O3 source). The warped metric is given by

$$\mathrm{d}s_{10}^2 = \tau^{-2}\mathrm{e}^{2A(y)}g_{\mu\nu}^4\mathrm{d}x^\mu\mathrm{d}x^\nu + \rho\mathrm{e}^{-2A(y)}g_{ij}^6\mathrm{d}y^i\mathrm{d}y^j \,, \tag{3.6}$$

Repeating the same steps as in [10] for O3 and O7 sources, one finds from the traced external Einstein equation and the F_5 Bianchi identity the following condition

$$\Box(\mathrm{e}^{4A} - \alpha) = \mathcal{R}_4 + \frac{\mathrm{e}^{2A}}{6Im\tau}\left|iG_3 - \star_6 G_3\right|^2 + \mathrm{e}^{-6A}\left|\partial\left(\mathrm{e}^{4A} - \alpha\right)\right|^2 \,. \tag{3.7}$$

If we integrate the equation on both sides over the internal manifold then we clearly find that $\mathcal{R}_4 > 0$ is impossible since the other 2 terms on the right hand are manifestly non-negative. This excludes any dS vacuum given the assumption for the F_5 field strength (11).

Let us therefore examine this assumption (3.5). Clearly for Calabi Yau spaces this assumption is necessary since there exist no non-trivial 1- or 5-cycles. But here we drop the Calabi-Yau assumption, such that one can in principle have

$$F_5 = (1 + \star)A \wedge \epsilon_4 \,, \tag{3.8}$$

where A is some cohomligical non-trivial one-form. In this case one cannot derive equation (3.7) to exclude dS solutions. However for O3 planes (3.8) is excluded since a non-trivial one-form would be projected out by the O3 involution. Hence the GKP argument also applies here and demonstrates that the minimal ingredients derived above are not sufficient since there do not exist tree-level dS solutions. This leaves O3/O7 models as the only possibility[6] (we already excluded O7).

"Pure flux" models. In this subsection we check whether the minimal ingredients can be satisfied in the simplified situation *that cycles thread by the field strengths and the cycles wrapped by the sources are closed but non-exact.*

Consider a field strength $F_p = \mathrm{d}C_{p-1}$. When we truncate all 4D vectors and 4D tensors, the dimensional reduction is

$$\hat{C}_q = \chi_i\Lambda_q^i \,, \qquad \hat{F}_p = \mathrm{d}\hat{C}_{p-1} + \Sigma_p \,, \tag{3.9}$$

where Σ_p are non-trivial elements of the p-th cohomology class $\Omega^P(M, \mathbb{R})$ of the internal manifold M. The χ^i are 4D scalar fields (the gauge potential moduli) and the Λ^i are a set

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[6]The combined O3 and O7 involution does not exclude (3.8) when the one-form A points in the direction of the O7 plane.

of p-forms on M, chosen such that the reduction corresponds to a consistent truncation. We define "pure flux" solutions as solutions for which we truncate all the gauge potential moduli: $\chi^i = 0$.

One needs to take into account the O-plane involutions to understand what kind of fluxes are allowed by the O-planes. An O-plane action is a combination of different involutions. There is always a target space involution σ and the world-sheet parity operation Ω, exchanging left and right movers. The fixed point set of the geometric involution σ defines the position of the O- plane. In some case one needs to add the involution $(-1)^{F_L}$, with F_L the left-moving fermion number. We have the following transformation properties:

$$\Omega: \quad + \quad \{\phi, g, C_1, C_2\}, \quad - \quad \{C_0, B_2, C_3, C_4\}, \tag{3.10}$$

$$(-1)^{F_L}: \quad + \quad \{\phi, g, B_2\}, \quad - \quad \{C_0, C_1, C_2, C_3, C_4\}. \tag{3.11}$$

It can be shown that, in order to divide out by symmetries of the string theory, the O-planes come with the following actions:

$$(-)^{F_L}\Omega\sigma \quad : \quad O3, O4, O6, O7, O8,$$

$$\Omega\sigma \quad : \quad O5, O9. \tag{3.12}$$

Hence to understand which degrees of freedom and which fluxes are allowed by the O-plane one multiplies the worldsheet involutions (3.12) for a certain field C (or flux F) and one considers how many legs of C (or flux F) are in the O-plane direction and how many are transversal. The latter is necessary to check the parity of the field, or flux, under σ. The total product should be even. Let us investigate this for the O4, O5 and O6 cases.

- *The O4 model*

  The Bianchi identity
  $$\mathrm{d}F_4 = H \wedge F_2 + \delta(O4), \tag{3.13}$$

  turns out problematic: F_2 has to thread a cycle with one leg in the O4 and another leg outside. If this flux is wedged with H we have a 5-form with at least one leg inside of the O4. This 5-form is hence of a different type then the 5-form distribution $\delta(O4)$, which has all legs outside of the O4 plane.

- *The O5 model*

  The Bianchi identities for F_3 and F_5
  $$\mathrm{d}F_5 = H \wedge F_3, \qquad \mathrm{d}F_3 = F_1 \wedge H + \delta(O5), \tag{3.14}$$

  where $\delta(O5)$ is a form distribution with 4 legs in the space transversal to the O5 plane. To evaluate these constraints we have to keep in mind that F_1, H and F_5 are odd under the O5 worldsheet operation involution and F_3 is even. Hence the F_1, H fluxes point in the transversal directions and the F_5 flux should have an odd number of legs along the transversal directions, whereas the F_3 should have an even number.

- *The O6 model*

  The F_2 Bianchi identity

  $$\mathrm{d}F_2 = mH + \delta(O6). \tag{3.15}$$

  demonstrates that H is needed to cancel the tadpole. Since H needs to thread a cycle transversal to the O6 it is the same form type as the form distribution of the O6 source, such that it can indeed cancel the tadpole. This is an attractive feature of these models.

Let us consider some examples. In case the internal space is a direct product of two 3-dimensional spaces $\mathcal{M}_3$ there is a straightforward way to define the O6 target space involution σ:

$$\sigma: \qquad (y_1, y_2, y_3, \bar{y}_1, \bar{y}_2, \bar{y}_3) \leftrightarrow (\bar{y}_1, \bar{y}_2, \bar{y}_3, y_1, y_2, y_3) \tag{3.16}$$

where the y and $\bar{y}$ represent coordinates on the 3D spaces. Then there is one O6 plane at the three-cycle spanned by the 3-surface, $y_i = \bar{y}_i$. Of course, there are other ways to define O6 planes, but this one is exceptionally easy.

In reference [5] some examples were studied where $\mathcal{M}_3$ are all 3D unimodular group manifolds and where $\mathcal{M}_3$ is the Weeks manifold (a compactification of the hyperboloid $\mathrm{SO}(3,1)/\mathrm{SO}(3)$). In the group manifold case it turned out that the other metric moduli, typical to group manifolds, have a runaway behavior in the $4ac/b^2$ expression, excluding any dS solutions. The Weeks manifold on the other hand has no moduli apart from ρ and τ. If we insist on not turning on massive shape moduli (that could be runaway) the possible fluxes are F_0, H and F_6 . When all these are turned on we have (ignoring all numerical factors)

$$\frac{4ac}{b^2} \propto \rho^2 + \rho^{-2} + \rho^0 + \rho^{-6} \tag{3.17}$$

and this shows that a dS can be found if one can tune the numerical factors, as turns out to be the case [5]. However this model fails to be a 10D solution since the F_4 equation of motion is not satisfied when $F_6 \neq 0$. If we put $F_6 = 0$ we loose the dS solution, since

$$\frac{4ac}{b^2} \propto \rho^2 + \rho^0. \tag{3.18}$$

A similar problem seems present in the model of [17] where there is also a non-zero F_6 flux.

It turns out that this happens more generically: one can find models for which one can stabilise the $4ac/b^2$ quantity, but if one then insists on satisfying all the 10D form equations of motion one finds exactly the terms needed for a solution to be forbidden. We illustrate this with one more example that captures the essentials. For that we take $\mathcal{M}_3 = \mathbb{H}_2 \times S_1$, where $\mathbb{H}_2$ is the compact 2D hyperboloid. The only modulus it has is the breathing mode. The metric Ansatz then is

$$\mathrm{d}s_{10}^2 = \tau^{-2}\mathrm{d}s_4^2 + \rho\left(\frac{1}{\phi}\mathrm{d}\mathbb{H}^2 + \phi^2\mathrm{d}y^2 + \frac{1}{\phi}\mathrm{d}\bar{\mathbb{H}}^2 + \phi^2\mathrm{d}\bar{y}^2\right). \tag{3.19}$$

So, there are 3 scalars, τ, ρ and ϕ, where the latter measures the relative size of the hyperboloid and the circle. The cycles that can be thread with fluxes, taking into account the parity of the O6 are:

$$H_3 : \qquad \epsilon_2 \wedge \mathrm{d}y - \bar{\epsilon}_2 \wedge \mathrm{d}\bar{y}\,, \tag{3.20}$$

$$F_2 : \qquad \epsilon_2 - \bar{\epsilon}_2\,, \qquad\qquad \mathrm{d}y \wedge \mathrm{d}\bar{y}\,, \tag{3.21}$$

$$F_4 : \qquad \epsilon_2 \wedge \bar{\epsilon}_2\,, \qquad\qquad (\epsilon_2 - \bar{\epsilon}_2) \wedge \mathrm{d}y \wedge \mathrm{d}\bar{y}\,, \tag{3.22}$$

where ϵ_2 is the volume element on $\mathbb{H}_2$. However upon using the 10D form equations one finds that the two F_2-fluxes need to vanish. The F_4 fluxes are not constrained and the H-flux is inversely proportional to the Romans mass F_0. If we ignore all numerical factors, we obtain the following expression

$$\frac{4ac}{b^2} \propto \phi\rho^2 + (\phi^5 + \phi^{-1})\rho^{-2} + (\phi^4 + \phi^{-2})\rho^{-4}\,. \tag{3.23}$$

It is not possible to stabilise ρ and ϕ at the same time. To see this clearly we make the following redefinition $\phi = \rho^{-2}\phi'$ and find

$$\frac{4ac}{b^2} \propto \phi' + (\phi'^5\rho^{-10} + \phi'^{-1}\rho^2)\rho^{-2} + (\phi'^4\rho^{-8} + \phi'^{-2}\rho^4)\rho^{-4}\,. \tag{3.24}$$

such that all powers in ρ are negative. One can readily check that when both F_2 fluxes are turned on, this problem disappears. So, it is really the information contained in the 10D form equations that spoil the putative dS solution.

4 O6 models on SU(3)-structure manifolds

In this section we reverse our strategy. Instead of investigating the scalar potential coming from a specific internal manifold with fluxes and then imposing the tadpole conditions, we consider a whole class of internal manifolds with an Ansatz for the fluxes that solves the 10D form equations from the outset.[7]

Since fluxes backreact the internal spaces to generalised Calabi-Yau spaces we take as a starting point a general class of SU(3)-structure manifolds defined by two torsion classes W_1 and W_2. Consider the canonical real two-form J and the complex three-form $\Omega = \Omega_R + i\Omega_I$ built out of the covariantly constant spinor on the internal manifold. We have the following characteristic equations

$$\mathrm{d}J = -\frac{3i}{2}W_1\Omega_R\,, \tag{4.1}$$

$$\mathrm{d}\Omega = W_1 J \wedge J + W_2 \wedge J\,, \tag{4.2}$$

where we assume that W_1 is an imaginary zero-form and W_2 is an imaginary two-form.

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[7]We stress that it is not necessary to investigate the 10D equations as long as one performs a consistent dimensional reduction, which we believe can be done for the models under consideration. However, we have found it easier to analyse the 10D equations instead of performing the reduction.

These kind of SU(3)-structure spaces have been shown to allow for supersymmetric AdS$_4$ solutions [3, 24, 25] with and without sources. Below we generalise the AdS Ansatz of [3, 24, 25] and check whether it can give rise to dS$_4$ solutions. For the readers' convenience we added appendix C that contains our IIA conventions and appendix D that contains useful formulae involving SU(3)-structures.

Our 10D Ansatz for the forms is

$$F_2 = \mathrm{e}^{-3\phi/4} f_1 J + i\mathrm{e}^{-3\phi/4} f_2 W_2 \,, \tag{4.3}$$

$$H = \mathrm{e}^{\phi/2} h \Omega_R \,, \qquad\qquad F_0 = \mathrm{e}^{-5\phi/4} m \,, \tag{4.4}$$

$$F_4 = \mathrm{e}^{-\phi/4} g_1 \epsilon_4 + \mathrm{e}^{-\phi/4} g_2 J \wedge J \,. \tag{4.5}$$

Concerning the O6 plane source we assume the same as in [3] that it is smeared and that it wraps the calibrated submanifold dual to Ω_R such that the Bianchi identity reads

$$\mathrm{d}F_2 = mH + \mu \Omega_R \,, \tag{4.6}$$

where in this convention positive μ implies net orientifold charge. The 10D Bianchi and form equations are solved if the flux parameters obey

$$g_1 h = -3i g_2 W_1 \,, \tag{4.7}$$

$$ih W_1 = 2 f_1 g_2 - g_1 g_2 + \frac{1}{2} m f_1 \,, \tag{4.8}$$

$$h = 2 f_2 g_2 - m f_2 \,, \tag{4.9}$$

$$f_2 \frac{|W_2|^2}{8} = mh + \frac{3i}{2} f_1 W_1 + \mathrm{e}^{3\phi/4} \mu \,, \tag{4.10}$$

and the following form equation is satisfied[8]

$$\mathrm{d}W_2 \propto \Omega_R \,. \tag{4.11}$$

From here on we just use the Bianchi identity (4.10) to determine the sign and the magnitude of μ. Of course, in an explicit model, the magnitude of the net orientifold charge cannot be chosen at will, since (i) the orientifold plane charges, just like D-brane charges, are quantized, and (ii) orientifold planes cannot be stacked like D-branes and their number is fixed through the number of $\mathbb{Z}_2$ involutions on the internal manifold.

For specific values of the flux parameters f_1, f_2, h, g_1, g_2 one obtains the supersymmetric AdS solutions of [3, 24]. These solutions have

$$f_1 = \frac{i}{4} W_1 \,, \qquad\qquad f_2 = 1 \,, \qquad\qquad h = -\frac{2m}{5} \,, \tag{4.12}$$

$$g_1 = 9 f_1 \,, \qquad\qquad g_2 = \frac{3m}{10} \,. \tag{4.13}$$

However, these ingredients are also sufficient to evade the usual dS no-go theorems. It is therefore interesting to understand whether there are other non-supersymmetric solutions in the 5-dimensional parameter-space (f_1, f_2, h, g_1, g_2).

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[8]One can prove that this assumption fixes the constant of proportionality to become $\mathrm{d}W_2 = -(i|W_2|^2/8)\,\Omega_R$.

The most constraining 10D equation is the internal Einstein equation ([C.6](#)). For the manifolds under consideration there exist explicit expressions for the Ricci tensor in terms of the forms J, Ω, W_2 [26, 27]:

$$\mathcal{R}_{mn} = -\frac{3i}{4}(\Omega_R)_n{}^{ps}\partial_{[p}(W_2)_{sm]} - \frac{1}{4}W_1(W_2)_{mr}J_n{}^r - \frac{1}{2}(W_2)_{mq}(W_2)_n{}^q + \frac{5}{4}g_{mn}|W_1|^2. \quad (4.14)$$

This clean expression implies we can verify in all generality the 10D equations of motion. The main clue to solve the 10D Einstein equations is the understanding of which tensors on both sides of the equation are independent. Clearly the traceless parts have to be equal. The problem divides into two cases

$$\text{case 1}: \qquad (W_2^2)_{ij} \neq \frac{W_2^2}{6}g_{ij} + i\alpha(JW_2)_{ij}, \quad (4.15)$$

$$\text{case 2}: \qquad (W_2^2)_{ij} = \frac{W_2^2}{6}g_{ij} + i\alpha(JW_2)_{ij}, \quad (4.16)$$

with α some real number different from zero.[9] Case 1 is the most general case and leads to the most restrictions. In case 2 the Einstein equations enforces less restrictive conditions and we will show that dS solutions are possible in this case.

The non-degenerate case. Let us first discuss case 1 and demonstrate that the only solutions are the supersymmetric AdS solutions constructed in [3, 24, 25]. If we just focus on the tensors different from g_{ij} in the internal Einstein equation we find two conditions from equating the coefficients in front of the W_{ij}^2 and $(JW)_{ij}$ tensors on both sides of the Einstein equation:

$$f_2 = \pm 1, \qquad -\frac{1}{4}W_1 = if_1f_2. \quad (4.17)$$

Combined with the equations ([4.7](#))–([4.10](#)) we uniquely find the known supersymmetric AdS solutions ([4.12](#)), ([4.13](#)). In fact, without analysing the Einstein equation, supersymmetry would immediately lead to these values for the fluxes and susy would guarantee that the Einstein and dilaton equations are solved. Since dS vacua are not supersymmetric there is more work in order to check when there is a solution.[10]

The degenerate case with $F_4 = 0$. Let us now consider case 2. The traceless part of the Einstein equations now imposes just one condition

$$(-f_2^2 + 1)\alpha = -2f_1f_2 + \frac{i}{2}W_1. \quad (4.18)$$

First we consider the simplified case where $F_4 = 0$. From here on we leave h and m free and solve all quantities in terms of these two flux numbers. Furthermore the ratio h/m is important enough to deserve a separate name

$$\beta = \frac{h}{m}. \quad (4.19)$$

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[9] In case 2 one can also verify that $\alpha \neq 0$. To show this note that J and W commute as matrices and therefore can be complex diagonalised at the same time. Using this as a starting point one finds that W^2 cannot be proportional to the metric when at the same time keeping JW traceless.

[10] However, recently it has been shown that some non-susy vacua have the same integrability properties as the susy vacua [28]. We did not pursue this possibility further.

Then the equations (4.7)–(4.9) imply

$$f_1 = 2\beta i W_1 \,, \qquad f_2 = -\beta \,, \tag{4.20}$$

The F_2 Bianchi identity (4.10) leads to

$$\frac{\mathrm{e}^{3\phi/4}\mu}{\beta m^2} = -1 - \frac{1}{m^2}\left(3|W_1|^2 + \frac{1}{8}|W_2|^2\right) \,. \tag{4.21}$$

From this we observe that without source we cannot have a solution and that $\beta > 0$ corresponds to net D6 charge and $\beta < 0$ to net O6 charge. In case we are interested in dS solutions we therefore need $\beta < 0$.

The remaining equations to verify are the traced internal Einstein equation and the dilaton equation, which are equivalent to the $\partial_\tau V = \partial_\rho V = 0$ equations

$$\partial_\rho V = 0: \qquad -V_R - 3V_H + 3V_0 + V_2 = 0 \,, \tag{4.22}$$

$$\partial_\tau V = 0: \qquad -2V_R - 2V_H - 4V_0 - 4V_2 - 3V_{O6/D6} = 0 \,, \tag{4.23}$$

where

$$V_R = -\frac{15}{2}|W_1|^2 + \frac{1}{4}|W_2|^2 \,, \qquad V_0 = \frac{m^2}{2} \,, \qquad V_H = 2h^2 \,, \tag{4.24}$$

$$V_2 = \frac{1}{4}\left(6f_1^2 + f_2^2|W_2|^2\right) \,, \qquad V_{O6} = -4\mu\mathrm{e}^{3\phi/4} \,. \tag{4.25}$$

In order to verify that we have a solution we must solve (4.22) and (4.23) for $|W_1|^2$ and $|W_2|^2$ and check when the expressions are positive. The solutions are

$$|W_1|^2 = \frac{-m^2}{81\beta}\left(5 + 16\beta - 20\beta^2 - 28\beta^3\right) \,, \tag{4.26}$$

$$|W_2|^2 = \frac{-2m^2}{27\,\beta\,(\beta+1)}\left(25 + 24\beta - 56\beta^2 + 192\beta^3 + 112\beta^4\right) \,. \tag{4.27}$$

Clearly both expressions are positive when β is negative and sufficiently close to zero. From the Bianchi identity we know that this also implies a net orientifold charge. In order to know what the sign of the 4D cosmological constant is one observes that equations (4.22) and (4.23) imply (only when $F_4 = 0$)

$$V = \frac{2}{3}(V_0 - V_H) \quad \Longrightarrow \quad V > 0: \quad \beta^2 < \frac{1}{4} \,. \tag{4.28}$$

Hence a small negative β is nicely consistent with a de Sitter solution! To understand what kind of solutions are possible we present some plots. In figure 1 we plot $|W_1|^2$ and $|W_2|^2$ as functions of β. A solution exists when both expressions are positive. In figure 2 we plot V and the two mass^2 eigenvalues in the ρ and τ directions as functions of β. From the figures we see that a value of β between roughly -2 and -1 gives rise to a non-supersymmetric AdS vacuum that is stable in the ρ and τ directions. We also note that we have dS vacua with a tachyonic direction for small negative values of β.
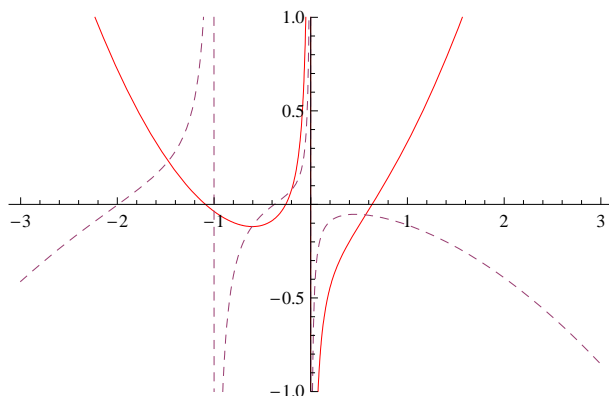
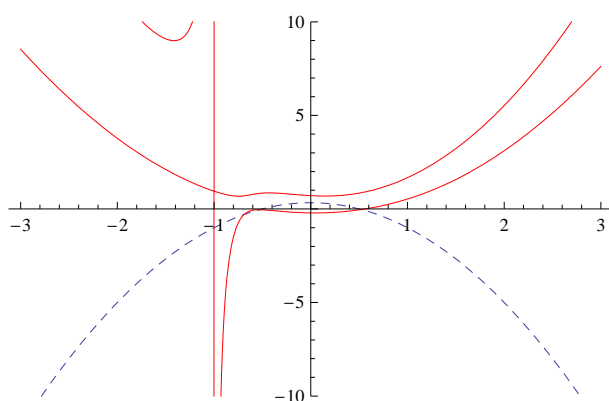**Figure 1**. $|W_1|^2$ and $10^{-2} \times |W_2|^2$ (dashed) as functions of β when $F_4 = 0$.

Figure 2. The two $10^{-1} \times$ mass^2 eigenvalues and V (dashed) as functions of β when $F_4 = 0$.

The degenerate case with $F_4 \neq 0$. In what follows it is useful to also define a new fraction

$$\gamma \equiv \frac{g_2}{m} \,. \tag{4.29}$$

We can solve f_1, f_2 and g_2 in terms of β, γ, m and the torsion classes as follows

$$f_2 = \frac{\beta}{2\gamma - 1} \,, \qquad g_1 = \frac{-3\gamma i W_1}{\beta} \,, \qquad f_1 = \frac{\beta - \frac{3\gamma^2}{\beta}}{\frac{1}{2} + 2\gamma} i W_1 \,. \tag{4.30}$$

Then the F_2 Bianchi identity (4.10) is given by

$$\frac{\mathrm{e}^{3\phi/4}\mu}{\beta} = -m^2 + \frac{|W_2|^2}{16\gamma - 8} - \frac{\left(3 - \frac{9\gamma^2}{\beta^2}\right)}{1 + 4\gamma}|W_1|^2 \,. \tag{4.31}$$

An interesting effect of non-zero γ is that the Bianchi identity can be satisfied with zero source $\mu = 0$. The contributions to the potential are now (V_R, V_{O6}, V_0, V_2 remain unaltered)
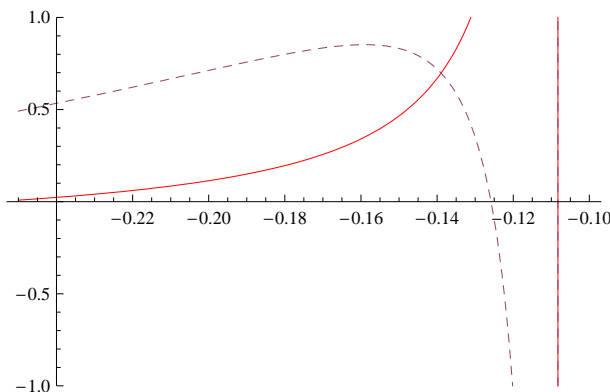
$$V_4 = 6g_2^2 \,, \qquad V_6 = \frac{1}{2}g_1^2 \,. \tag{4.32}$$

Figure 3. $|W_1|^2$ and $10^{-1} \times |W_2|^2$ (dashed) as functions of β for $\gamma = 0.1$.
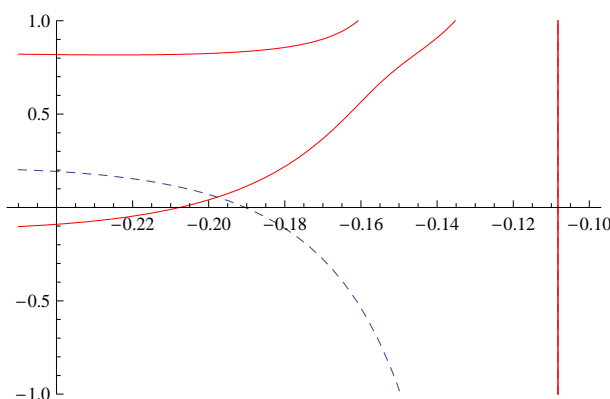

Figure 4. The two $10^{-1} \times$ mass^2 eigenvalues and V (dashed) as functions of β for $\gamma = 0.1$.

Having established this we can repeat the same kind of analysis as above. One rewrites the $\partial_\rho V = \partial_\tau V = 0$ equations in terms of $\beta, \gamma, m, |W_1|^2, |W_2|^2$ and checks when there exists solutions, i.e., when the solutions for $|W_{1,2}|^2$ in terms of (β, γ, m) are positive.[11] Below we present plots of $|W_{1,2}|^2$ in terms of β for $\gamma = 0.1$. From figure 3 and 4 we see that dS solutions, stable in the ρ, τ-directions exist for β between about -0.207 and -0.190. Note that while a critical dS is easy to achieve, there is just a tiny little window available for a solution stable in the ρ, τ-directions. From figure 5 it can be seen that these solutions have a net orientifold charge.

From figure 5 we can see that, if β is chosen near -0.13 the solution has a vanishing O6/D6 charge. At this value of β both the mass matrix eigenvalues are positive as can be seen from figure 4. So, we get AdS solutions with vanishing charge that are stable in the ρ, τ-directions.

The scales. Note that in the previous we have absorbed ρ and τ in the various fluxes such that it did not appear explicitly in the equations (4.24), (4.25) and (4.32). Note that

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[11]It turns out that m^2 just sets the overall scale and one can therefore just take $m^2 = 1$. Then one is left with β, γ.
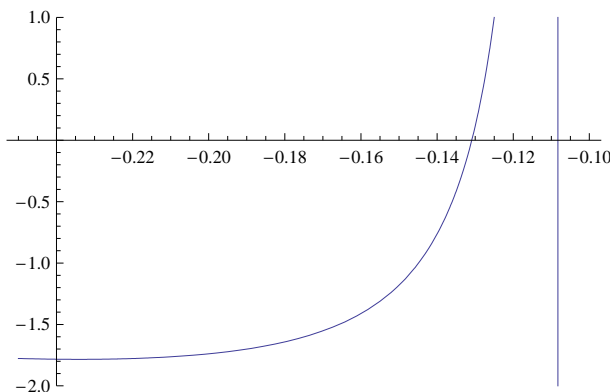
Figure 5. V_{O6} as a function of β for $\gamma = 0.1$.

we can therefore choose it at will by rescaling the various fluxes (and μ). This implies that we can make the solution as weakly coupled as we want and choose the volume such that we can neglect α' corrections and perhaps still have a decoupling of KK modes [3]. However there is a danger since we also have to rescale μ, but, as we explained before, the number of O-planes is not a free parameter. Furthermore, scaling of fluxes is also potentially dangerous because of quantisation. So, it remains to be seen whether a given explicit model fulfills the right conditions.

If we reinstate the dependence of ρ and τ in the equations we can plot the potential in function of ρ and τ. This we have done in figure 6 for $\beta = -0.2$. We have chosen ρ and τ such that the dS minimum derived above is at $\rho = \tau = 1$. We can clearly see the minimum, and in addition an inflexion point near $(\rho, \tau) = (1.08, 1.25)$ acting as a barrier against a deeper drop in the potential towards the upper right in the picture. This is qualitatively the same kind of behaviour as in KKLT [16] and suggests a dS vacua non-perturbatively unstable against tunneling to a lower energy. However, one needs to be very careful when drawing these kinds of conclusions. The only critical point in figure 6 that we have actually proven to be a solution to the 10D equations of motion is the minimum at $(\rho, \tau) = (1, 1)$. Any other critical points generated by moving off in the (ρ, τ)-plane are likely not to be full solutions.

The coset geometries. So far we have not realised explicit geometries for these torsion classes. In total we have 2 conditions on the torsion classes.

1. $dW_2 \propto \Omega_R$,

2. $W_{ij}^2 = \frac{1}{6} W_2^2 g_{ij} + i\alpha(JW)_{ij}$,

where the Einstein equations dictate that α is related to the fluxes as follows

$$\frac{\alpha}{iW_1} = \left(\frac{2\gamma - 1}{2 + 8\gamma} \right) \frac{8(3\gamma^2 - \beta^2) + (2\gamma - 1)(1 + 4\gamma)}{(2\gamma - 1)^2 - \beta^2} \,. \tag{4.33}$$

We will now check these conditions for the coset geometries and the Iwasawa manifold discussed in [3].[12]

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[12]Another set of explicit SU(3)- structure manifolds appeared in [29]. These spaces have $W_1 = 0$ and $W_2 \neq 0$. We have verified that this does not allow the dS solutions we have considered.
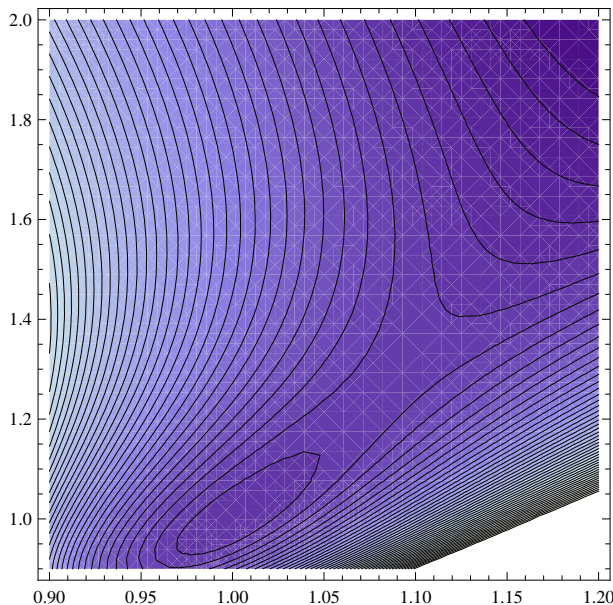
Figure 6. V as a function of ρ and τ for $\beta = -0.2$ and $\gamma = 0.1$.

One finds that the only examples that can fulfill $dW_2 \sim \Omega_R$ and the degeneracy condition are $\mathrm{Sp}(2)/S(\mathrm{U}(2) \times \mathrm{U}(1))$ and $\mathrm{SU}(3)/\mathrm{U}(1) \times \mathrm{U}(1)$. It turns out $\mathrm{Sp}(2)/S(\mathrm{U}(2) \times \mathrm{U}(1))$ is a special subcase of $\mathrm{SU}(3)/\mathrm{U}(1) \times \mathrm{U}(1)$ when some moduli are fixed; so we only discuss the coset $\mathrm{SU}(3)/\mathrm{U}(1) \times \mathrm{U}(1)$. According to [3, 30] we have

$$J = -ae^{12} + be^{34} - ce^{56} \tag{4.34}$$

$$\Omega = d\left(\left(e^{245} + e^{135} + e^{146} - e^{236}\right) + i\left(e^{235} + e^{136} + e^{246} - e^{145}\right)\right), \tag{4.35}$$

where the e^i are the Cartan-Maurer forms. The metric is diagonal with respect to the Cartan-Maurer forms and is given by

$$g = \begin{pmatrix} a & & & & & \\ & a & & & & \\ & & b & & & \\ & & & b & & \\ & & & & c & \\ & & & & & c \end{pmatrix}. \tag{4.36}$$

For convenience we introduce the notation $g = (a, b, c)$. We furthermore have

$$W_1 = \frac{i}{3} \frac{a + b + c}{\sqrt{abc}}, \tag{4.37}$$

$$W_2 = -\frac{2i}{3\sqrt{abc}} \left(a\left(2a - b - c\right) e^{12} + b\left(a - 2b + c\right) e^{34} + c\left(-a - b + 2c\right) e^{56}\right). \tag{4.38}$$

– 16 –

From these expressions we find

$$|W_2|^2 = \frac{16}{3abc}\Big(a^2 + b^2 + c^2 - (ab + ac + bc)\Big), \tag{4.39}$$

$$(W_2^2)_{nm} = -\frac{4}{9abc}\Big(a\,(2a - b - c)^2\,, b\,(2b - a - c)^2\,, c\,(2c - a - b)^2\Big) \tag{4.40}$$

$$(JW_2)_{mn} = \frac{2i}{3\sqrt{abc}}\Big(a\,(2a - b - c)\,, b\,(2b - a - c)\,, c\,(2c - a - b)\Big). \tag{4.41}$$

In general $(JW_2)_{mn}$ and $(W_2^2)_{mn} - \frac{1}{6}g_{nm}W_2^2$ are not parallel to each other, but at, e.g.,
$a = b$ we find

$$\alpha = \frac{2(c - a)}{3a\sqrt{c}} \tag{4.42}$$

So, we should look for solutions with this value for α and

$$|W_1|^2 = \frac{(2a + c)^2}{9a^2 c}\,, \qquad |W_2|^2 = \frac{16}{3a^2 c}\,(a - c)^2\,, \tag{4.43}$$

where $a, c > 0$.

We have been able to find such solutions corresponding to new, non-supersymmetric
AdS vacua. For instance, with $\gamma = 0.1$ we find two solutions, both stable in the ρ and τ
directions (we take $m^2 = 1$):

$$a \approx 1.355\,, \qquad c \approx 0.5889\,, \qquad \beta \approx -0.129\,. \tag{4.44}$$

This solution has net D-brane charge (as can be verified using the plots). The other solution
has net O6 charge

$$a \approx 1.7625\,, \qquad c \approx 0.7718\,, \qquad \beta \approx 0.126\,. \tag{4.45}$$

Such non-supersymmetric AdS4 vacua will be studied in more detail in [31].

We have not been able to find any dS solutions for this coset, in agreement with the
results of [4].

The Iwasawa manifold. There is one extra example discussed [3] that can satisfy the
degeneracy condition and $dW_2 \propto \Omega_R$. This is the Iwasawa manifold. In Cartan-Maurer
basis the metric is given by

$$g = \left(1, 1, y^2\right)\,. \tag{4.46}$$

with y some fixed number. Furthermore

$$J = e^{12} + e^{34} - y^2 e^{56}\,, \tag{4.47}$$

$$W_2 = -\frac{4iy}{3}\left(e^{12} + e^{34} + 2y^2 e^{56}\right)\,, \tag{4.48}$$

$$W_1 = -\frac{2iy}{3}\,. \tag{4.49}$$

From this we have

$$\alpha = \frac{4y}{3}\,, \qquad |W_1|^2 = \frac{4y^2}{9}\,, \qquad |W_2|^2 = \frac{64y^2}{3}\,. \tag{4.50}$$

We have not been able to find other vacuum solutions apart from the susy AdS ones. So
for the Nilmanifold it is possible to have the susy choice for the fluxes and, at the same
time, have the degeneracy in the tensors JW_2 and W_2^2.

5 Discussion

In this paper we have investigated on general grounds the conditions for the existence of classical de Sitter solutions in string theory. We also went further by analysing specific Type IIA O6 constructions. The simplest models, in which the fluxes are closed and non-exact, generically have moduli directions for which the potential has no stationary dS point (not even an unstable one). However, using SU(3)-structure solutions as a testbed for models that have different kinds of fluxes, we were able to find a simple set of conditions on the torsion classes in order for specific de Sitter solutions to exist. We explicitly verified these conditions for the coset geometries and found that these conditions could almost be satisfied but not quite. This we take as an indication that our conditions, though non-trivial, are not impossible to be realized. For the coset geometries that had the almost correct form of the torsion classes we were able to find new non-supersymmetric AdS solutions.

Concerning stability we have only investigated the masses of the ρ and τ scalars. The general moduli structure of these generalised Calabi Yau spaces is not understood and can only be studied when there exist an explicit geometry realising our conditions on the torsion classes. However for the new AdS solutions we have found in the $SU(3)/U(1) \times U(1)$ coset construction, it should be possible to use the effective theory developed in [3] (for a consistent subset of the degrees of freedom) to study the stability.

Once an explicit geometry for the dS solution satisfying our conditions is found, it is important to study the charge (and flux) quantisation since the O6 charge depends on the involutions present in the explicit geometry. While our analysis relies on the smearing of the O-planes, they should be understood, in a fully microscopic construction of our dS solutions, as a localized source whose singularity admits a stringy resolution. We consider it as an important avenue for further research to understand the effect of the backreaction of the sources defined in this microscopic manner.

While SUSY AdS can be argued to be quite generic, dS solutions to the equations of motion require fluke alignment of various contributions to the internal Einstein equations. It therefore seems likely that dS solutions should be regarded as accidental from a landscape point of view. If one, furthermore, requires perturbative stability in all directions, it might become exceedingly difficult to find actual examples (see e.g. [32]). While our analysis has been purely perturbative, there is no reason to expect that the difficulties would go away in a non-perturbative setting. Unfortunately, the presently available methods do not allow for a detailed analysis of the non-perturbative case.

Given a critical point there is really no reason to expect a minimum along a particular direction in moduli space. The critical point might as well be a maximum or an inflexion point, and one might argue that the chances for a given critical point to be a minimum in one direction is only around $1/2$. If the dimensionality of the moduli space is N, then the fraction of critical dS points that actually are minima is down by a factor 2^{-N}. With N of the order of a few hundred, this reduction with respect to the total number of critical dS points in the landscape can easily be of the same order, or even exceed, the expected 10^{-120} from the smallness of the observed cosmological constant. One can therefore argue that the existence of a perturbatively stable dS vacua, is at least as severe a finetuning as the size of

the cosmological constant itself. It is in fact far from obvious that there are any candidate vacua left in the landscape at all. Hence, it is reasonable to investigate whether perturbatively unstable dS critical points can work from a phenomenological point of view [33].

Finally we like to mention some interesting directions for further research. One obvious direction is to find explicit geometries that satisfy our conditions on the torsion classes needed for our simple de Sitter solutions. If this can be done one can study the stability of the solutions and the effect of the charge and flux quantisation. On the other hand, when one considers explicit geometries one can also allow more general fluxes then the one we considered (those given by Ω, J and W_2) as was done for instance in [4]. In general this is a hard problem, but can be done if one can systematically scan the scalar potential in these IIA orientifold models (see e.g. [3, 34, 35]) for critical points. In some interesting cases (like for twisted tori), the effective theory is $\mathcal{N} = 4$ gauged supergravities [36, 37] which facilitate a systematic scanning for de Sitter critical points [38, 39].

Acknowledgments

A Form conventions and useful formulae

A p-form A_p in components is given by

$$A_p = \frac{1}{p!} A_{\mu_1 \ldots \mu_p} \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_p} \,. \tag{A.1}$$

Forms obey the following algebra

$$A_p \wedge B_q = (-)^{pq} B_q \wedge A_p \,. \tag{A.2}$$

The exterior derivative is defined via

$$\mathrm{d}A_p = \frac{1}{p!} \partial_{[\nu} A_{\mu_1 \ldots \mu_p]} \mathrm{d}x^{\nu} \wedge \mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_p} \,, \tag{A.3}$$

and obeys the Leibniz rule

$$\mathrm{d}(A_p \wedge B_q) = \mathrm{d}A_p \wedge B_q + (-)^p A_p \wedge \mathrm{d}B_q \,. \tag{A.4}$$

In D dimensions we define the epsilon *symbol* $\varepsilon_{\mu_1\mu_2...\mu_p}$ via

$$\varepsilon_{01...D-1} = 1\,, \tag{A.5}$$

and it is antisymmetric in all indices $\varepsilon_{[\mu_1\mu_2...\mu_p]} = \varepsilon_{\mu_1\mu_2...\mu_p}$. From the epsilon symbol we define the epsilon *tensor* $\varepsilon_{\mu_1\mu_2...\mu_p}$ via

$$\epsilon_{\mu_1\mu_2...\mu_p} = \sqrt{|g|}\varepsilon_{\mu_1\mu_2...\mu_p}\,. \tag{A.6}$$

Contractions of the epsilon tensor (and symbol) obey the following relations

$$\epsilon_{\mu_1\mu_2...\mu_q\mu_{q+1}...\mu_D}\epsilon^{\mu_1\mu_2...\mu_q\nu_{q+1}...\nu_D} = (-)^t q!(D-q)!\,\delta^{[\nu_{q+1}}_{[\mu_{q+1}}...\delta^{\nu_D]}_{\mu_D]}\,, \tag{A.7}$$

where t stands for the number of timelike dimensions of the D-dimensional space. The Hodge operator $\star$ maps p-forms into $(D-p)$-forms. We define $\star$ on the coordinate p-forms and by linearity it is defined on all forms

$$\star\,(\mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_p}) = \frac{1}{(D-p)!}\epsilon_{\nu_1...\nu_{D-p}}{}^{\mu_1...\mu_p}\,\mathrm{d}x^{\nu_1} \wedge \ldots \wedge \mathrm{d}x^{\nu_{D-p}}\,. \tag{A.8}$$

The $\star$ operation has the following properties

$$\star\,A_p \wedge B_p = \star B_p \wedge A_p = \frac{1}{p!}A_{\mu_1...\mu_p}B^{\mu_1...\mu_p}\star 1\,, \tag{A.9}$$

$$\star\star A_p = (-)^{p(D-p)+t}A_p\,. \tag{A.10}$$

Useful identities are

$$\mathrm{d}x^{\mu_1} \wedge \ldots \wedge \mathrm{d}x^{\mu_D} = (-)^t\varepsilon^{\mu_1...\mu_D}\mathrm{d}x^0 \wedge \ldots \wedge \mathrm{d}x^{D-1}\,, \tag{A.11}$$

$$\star 1 = \sqrt{|g|}\mathrm{d}x^0 \wedge \ldots \wedge \mathrm{d}x^{D-1}\,. \tag{A.12}$$

As an application of these conventions one has

$$\star_{10}\,(A_p \wedge B_q) = (-1)^{p(6-q)}\star_4 A_p \wedge \star_6 B_q\,, \tag{A.13}$$

where A is a form in four-dimensional spacetime and B_q is a form on the internal six-dimensional space.

For a metric of the form

$$\mathrm{d}s_{10}^2 = \tau^{-2}\mathrm{e}^{2\alpha A(y)}g_{\mu\nu}^4\mathrm{d}x^\mu\mathrm{d}x^\nu + \rho\mathrm{e}^{2\beta A(y)}g_{ij}^6\mathrm{d}y^i\mathrm{d}y^j\,, \tag{A.14}$$

The Ricci tensor is (assuming constant τ and ρ)

$$\begin{aligned}
\mathcal{R}_{\mu\nu}^{10} &= R_{\mu\nu}(g^4) - 4(\alpha^2 + \alpha\beta)\mathrm{e}^{2(\alpha-\beta)A(y)}(\partial A)^2\tau^{-2}\rho^{-1}g_{\mu\nu}^4\,, \\
&\quad -\alpha\mathrm{e}^{2(\alpha-\beta)A}\tau^{-2}\rho^{-1}g_{\mu\nu}^4\Box A\,, \tag{A.15}
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_{ij}^{10} &= R_{ij}(g^6) - 4(\alpha^2 + \alpha\beta)(\partial A)^2 g_{ij}^6 + 4(\beta^2 - \alpha^2 + 2\alpha\beta)\partial_i A\partial_j A \\
&\quad -(6\alpha + 4\beta)\nabla_i\partial_j A - \beta g_{ij}^6\Box A\,. \tag{A.16}
\end{aligned}$$

B 10D Einstein and dilaton equation

The 10D action is (where we have put $\kappa_{10}^2 = 1/2$)

$$\int \sqrt{g} \left\{ R - \frac{1}{2}(\partial\phi)^2 - \sum_n \frac{1}{2\,n!} e^{a_n\phi} F_n^2 \right\} + S_{\text{loc}} \,, \tag{B.1}$$

where Σ_n represents the sum over all the field strengths and the numbers a_n are given by

$$a_n^{\text{RR}} = \frac{5-n}{2} \,, \qquad a_3^{\text{NS}} = -1 \,. \tag{B.2}$$

In IIA the RR field strengths are F_0, F_2, F_4. When space-filling F_4 flux is considered we will define it using F_6. In IIB the RR fields strengths are F_1, F_3, F_5, where F_5 is assumed to be self-dual. The source action is

$$S_{\text{loc}} = - \int_{p+1} T_p \sqrt{|g|} + \mu_p \int_{p+1} C_{p+1} \,, \qquad T_p = \pm\mu_p e^{(p-3)\phi/4} \,. \tag{B.3}$$

The Einstein equation is (for ϕ constant)

$$R_{ab} = \sum_n \left(-\frac{n-1}{16n!} g_{ab} e^{a_n\phi} F_n^2 + \frac{1}{2(n-1)!} e^{a_n\phi} (F_n)_{ab}^2 \right) + \frac{1}{2} \left(T_{ab}^{\text{loc}} - \frac{1}{8} g_{ab} T^{\text{loc}} \right) \tag{B.4}$$

where the local stress tensor is given by

$$T_{\mu\nu}^{\text{loc}} = -T_p\, g_{\mu\nu}\, \delta(\Sigma) \,, \qquad T_{ij} = -T_p\, \Pi_{ij}\, \delta(\Sigma) \,, \tag{B.5}$$

throughout a, b are 10D indices, i, j are internal and $\mu\nu$ are external. Π_{ij} is the projector on the cycle wrapped by the source. In the smeared limit (which is considered when $p > 3$) we have

$$\delta(\Sigma) \to 1 \,, \qquad \Pi_{ij} \to \frac{p-3}{6}\, g_{ij} \,. \tag{B.6}$$

Taking the trace over the internal indices and integrating over the 6D space one finds ($V_p = T_p$):

$$-V_R = \sum_n \frac{(n+3)}{4} V_n + \frac{1}{8}(15-p)V_p \,. \tag{B.7}$$

The 10D dilaton equation is

$$\Box\phi = 0 = \sum_q \frac{a_n}{2n!} e^{a_n\phi} F_n^2 - \frac{p-3}{4} e^{(p-3)\phi/4} \mu_p\, \delta(\Sigma) \,, \tag{B.8}$$

from which we have

$$\sum_n a_n V_n + \frac{p-3}{4} V_p = 0 \,. \tag{B.9}$$

From the expression for the effective potential we find:

$$\partial_\rho V = 0: \qquad -V_R - 3V_H + \sum_q (3-q)V_q + \frac{(p-6)}{2} V_p = 0 \,, \tag{B.10}$$

$$\partial_\tau V = 0: \qquad -2V_R - 2V_H - 4\sum_q V_q - 3V_p = 0 \,. \tag{B.11}$$

where q runs over the RR field strengths. We notice that (B.7) can be found from summing $2/3$ times the first equation with the second equation. Equation (B.9) can be obtained from summing -2 times the first equation with the second equation.

The trace of the Einstein equation over the external indices just sets the value of the cosmological constant. This can best be seen using the ordinary Einstein equation

$$G_{\mu\nu} = \sum_n \frac{1}{n!} e^{a_n \phi} \left(n \, (F_n^2)_{\mu\nu} - \frac{1}{2} g_{\mu\nu} F_n^2 \right) + \frac{1}{2} T_{\mu\nu}^{\text{local}} \,. \tag{B.12}$$

When we take indices in the 4D spacetime we have[13] $(F_n^2)_{\mu\nu} = 0$. When we take the trace over the 4D indices and remember that using $R_{10} = R_4 + R_6$ and $R_4 = 2V$ we recover the definition of V

$$V = V_R + \sum_n V_n + V_p \,. \tag{B.13}$$

C IIA SUGRA

The form equations of motion are

$$d(\star e^{3\phi/2} F_2) + e^{\phi/2} \star F_4 \wedge H = 0 \,, \quad \text{(C.1)}$$

$$d(\star e^{\phi/2} F_4) - F_4 \wedge H = 0 \,, \quad \text{(C.2)}$$

$$d(\star e^{-\phi} H) + e^{\phi/2} \star F_4 \wedge F_2 - \frac{1}{2} F_4 \wedge F_4 + F_0 e^{3\phi/2} \star F_2 = 0 \,, \quad \text{(C.3)}$$

$$d \star d\phi - \frac{1}{4} e^{\phi/2} \star F_4 \wedge F_4 + \frac{1}{2} e^{-\phi} \star H \wedge H - \frac{3}{4} e^{3\phi/2} \star F_2 \wedge F_2 - \frac{5}{4} e^{5\phi/2} \star F_0 \wedge F_0 = 0 \,, \quad \text{(C.4)}$$

where F_0 is the Romans' mass. The Bianchi identities read

$$dH_3 = 0 \,, \qquad dF_2 = F_0 H \,, \qquad dF_4 = F_2 \wedge H_3 \,. \tag{C.5}$$

The Einstein equation is given by

$$0 = \mathcal{R}_{MN} - \frac{1}{2} \partial_M \phi \partial_N \phi - \frac{1}{12} e^{\phi/2} F_{MPQR} F_N^{PQR} + \frac{1}{128} e^{\phi/2} g_{MN} F_4^2 - \frac{1}{4} e^{-\phi} H_{MPQ} H_N^{PQ}$$

$$+ \frac{1}{48} e^{-\phi} g_{MN} H^2 - \frac{1}{2} e^{3\phi/2} F_{MP} F_N^P + \frac{1}{32} e^{3\phi/2} g_{MN} F_2^2 - \frac{1}{16} g_{MN} e^{5\phi/2} F_0^2 \,. \tag{C.6}$$

D SU(3)-structure equations

Fluxes in IIA SUGRA lead to SU(3)-structures as can be derived from the existence criterium of a spinor on the internal manifold. Out of the spinor bilinears one can define a real two form J and an imaginary self-dual three form Ω [24]. These forms satisfy many

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[13]in IIA with space filling F_4 we replace the space-filling component by F_6. In IIB with non-zero F_5 this term is non-zero but if one defines V_5 with an extra factor of $1/4$ the expressions match.

relations and we list those that are not presented in the main text and which are necessary for the computations presented in this paper:

$$\star_6 \Omega = -i\Omega\,, \qquad\qquad \star_6 J = \frac{1}{2}\,J \wedge J\,, \qquad (\text{D.1})$$

$$\Omega \wedge \Omega^* = \frac{4i}{3}\,J \wedge J \wedge J\,, \qquad\qquad J \wedge J \wedge J = 6\epsilon_6\,, \qquad (\text{D.2})$$

$$\Omega \wedge J = 0\,, \qquad\qquad W_2 \wedge J \wedge J = 0\,, \qquad (\text{D.3})$$

$$W_2 \wedge \Omega = 0\,, \qquad\qquad \star_6 W_2 = -J \wedge W_2\,, \qquad (\text{D.4})$$

$$J_{mn} W_2^{mn} = 0\,, \qquad\qquad J_m{}^n J_p{}^q (W_2)_{nq} = (W_2)_{mp}\,, \qquad (\text{D.5})$$

$$(\Omega_R)^2_{ab} = (\Omega_I)^2_{ab} = 4g_{ab}\,, \qquad\qquad J^2_{ab} = g_{ab}\,. \qquad (\text{D.6})$$

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